# Simple Schemes for Parallel Acquisition of Spreading Sequences in DS/SS Systems

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*Abstract*—**Suboptimal parallel schemes for the acquisition of spreading sequences in chip-asynchronous spread-spectrum systems are considered. These acquisition schemes try to estimate the unknown delay of the received signal with respect to a locally generated spreading code. Two schemes are presented which are considerably simpler to implement than the optimal estimator, and an exact formula is derived for the error probability of one of the schemes. Numerical results show that the performance of the suboptimal estimators is comparable to that of the optimal estimator.**

## I. Introduction

In a direct sequence spread-spectrum (DS/SS) communications system, in order for the receiver to demodulate the received signal, it must first synchronize its locally generated code sequence to the code sequence in the received signal. The synchronization process is divided into the two stages of acquisition and tracking [1]. Acquisition refers to the coarse synchronization of the received sequence and locally generated sequence to within some fraction of the chip duration of the code sequence. Once acquisition has been accomplished, a code tracking loop is employed to achieve fine alignment of the two sequences and maintain that alignment. Because communication cannot take place before acquisition has been achieved, the development of quick and effective acquisition schemes is important. We consider the acquisition process for the simplified model of the DS/SS system in which we assume that no data modulation is present and that carrier synchronization has been achieved.

Let the code sequence $\{c\} = (\ldots, c_0, c_1, c_2, \ldots)$ denote a PN sequence of period $N$, with $c_j \in \{-1, 1\}$, where by PN sequence we mean a binary maximal-length shift register sequence. Let $c(t) = \sum_{j=-\infty}^{\infty} c_j \Pi_{T_c}(t - jT_c)$ denote a pulse train, where $\Pi_{T_c}(\cdot)$ is a rectangular pulse function of duration $T_c$ centered at $T_c/2$. This pulse train modulates an RF carrier to produce the spreading signal $c(t)\cos(\omega_0 t + \phi)$. The receiver input corresponding to such a transmitted signal is

$$r(t) = \sqrt{2V} c(t + \delta T_c) \cos(\omega_0 t + \theta) + n(t),$$

where $V$ is the received signal power, $\delta T_c$ is the unknown time shift, $\theta = \phi + \omega_0 \delta T_c$ is the RF phase of the carrier, and $n(t)$ is additive white Gaussian noise with two-sided power spectral density $\eta_0/2$. We assume that the RF carrier has been completely acquired, so that the receiver is perfectly synchronized to the carrier in both frequency and phase. Thus it suffices to treat the receiver input as

$$r(t) = \sqrt{V} c(t + \delta T_c) + n(t).$$

Note that we have simplified the model by not including any data modulation on the signal. Many DS/SS transmissions include a preliminary training period during which only the phase-coded RF carrier is transmitted [1].

The acquisition problem is that of finding an estimate $\delta_{est} T_c$ of the unknown time shift $\delta T_c$ such that $\delta T_c - \delta_{est} T_c$ is within the pull-in range of the code tracking loop. Since the spreading code has period $NT_c$, we can assume that $\delta \in [0, N)$. Therefore, the signal is said to be acquired if

$$\min\{|\delta - \delta_{est}|, N - |\delta - \delta_{est}|\} \le \zeta$$

for some specified $\zeta$ corresponding to the pull-in range of the code tracking loop. The results in this paper are given for $\zeta = \frac{1}{2}$.

Chawla and Sarwate [2] presented several schemes for the parallel acquisition of PN sequences. Unfortunately, there are shortcomings to these schemes. The optimal estimator $\mathcal{S}_{opt}$ of [2] requires large amounts of computation, and its performance cannot be analyzed easily. On the other hand, the locally optimal estimator $\mathcal{S}_{lo}$ is easy to implement and analyze, but has quite poor performance [2]. In this paper, we present suboptimum estimators that are easy to implement and yet perform well. One of our estimators is essentially a hybrid of the maximum likelihood estimator presented in [2] and the optimal estimator. The performance of this simple estimator can be analyzed exactly, and the results provide bounds and approximations for the performance of the optimal estimator.

## II. Optimal Estimator

In a parallel acquisition scheme, the receiver computes (in parallel) the correlation of the received signal with the $N$ phases of the spreading sequence and estimates $\delta$ from the correlation values. Suppose $\delta = k + \epsilon$, where

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$k \in \{0, \ldots, N-1\}$ and $\epsilon \in [0,1)$. As shown in [2], the $N$ observations may be taken to be independent unit variance Gaussian random variables with $E[X_k] = (1-\epsilon)\mu$, $E[X_{k+1}] = \epsilon\mu$, and $E[X_i] = 0$ otherwise, where $\mu = [2VT_c(N+1)/\eta_0]^{1/2}$ is a measure of the signal-to-noise ratio (SNR). Thus, the conditional joint density function of $\mathbf{X}$ given $\delta = k + \epsilon$ is

$$h_{\mathbf{X}|\delta}(\mathbf{x}|k+\epsilon) = \phi(x_k - (1-\epsilon)\mu)\phi(x_{k+1} - \epsilon\mu) \prod_{\substack{i=0 \\ i \neq k,k+1}}^{N-1} \phi(x_i) \tag{1}$$

where $\phi(\cdot)$ denotes the unit Gaussian density function.

The optimal estimator maximizes the *a posteriori* probability that $\delta$ lies in an interval of width one centered at the estimate $\hat{\delta}$. Under the assumption that $\delta$ is uniformly distributed over $[0, N)$, the optimum estimate is given by

$$\delta_{opt} = \arg \max_{\hat{\delta} \in [0,N)} \int_{\hat{\delta}-1/2}^{\hat{\delta}+1/2} h_{\mathbf{X}|\delta}(\mathbf{x}|u)du. \tag{2}$$

As shown in [2], for $\hat{\delta} \in [l-1/2, l+1/2]$, (where $0 \leq l \leq N-1$), the integral in (2) has value

$$\begin{aligned} I(\hat{\delta}) = {} & R(\mathbf{x}, l-1, \mu)\left[\Phi((x_{l-1} - x_l + \mu)/\sqrt{2})) - \right. \\ & \left. \Phi((x_{l-1} - x_l + 2\mu(\hat{\delta} - l))/\sqrt{2})\right] \\ & + R(\mathbf{x}, l, \mu)\left[\Phi((x_l - x_{l+1} + 2\mu(\hat{\delta} - l))/\sqrt{2}) - \right. \\ & \left. \Phi((x_l - x_{l+1} - \mu)/\sqrt{2})\right], \end{aligned} \tag{3}$$

where $\Phi(\cdot)$ is the unit Gaussian distribution function and

$$R(\mathbf{x}, l, \mu) = \phi([x_l + x_{l+1} - \mu]/\sqrt{2}) \prod_{i \neq l,l+1} \phi(x_i)/\sqrt{2}\mu.$$

The algorithm $\mathcal{S}_{opt}$ for finding $\delta_{opt}$ is as follows [2]:

For each $0 \leq l \leq N-1$, if $x_l > \max\{x_{l-1}, x_{l+1}\}$, there is a local maximum of the function $I(\hat{\delta})$ in the interval $(l - \frac{1}{2}, l + \frac{1}{2})$ at

$$\hat{\delta}_l = l + \frac{x_{l+1} - x_{l-1}}{2(2x_l - x_{l-1} - x_{l+1})}. \tag{4}$$

Compute $I(\hat{\delta})$ at each local maximum and find the global maximum of these values. $\delta_{opt}$ is the location of this global maximum. (Here, and throughout the paper, we ignore the possibility that $x_l = x_{l+1}$ since these events have zero probability.)

Although $\mathcal{S}_{opt}$ minimizes the average error probability over all values of $\epsilon$, computation of the decision statistic $I(\hat{\delta})$ requires the evaluation of exponential and error functions. Since there are roughly $N/3$ local maxima, and it is necessary to compute (3) at each one, this scheme is computationally intensive. In addition, the error probability is very difficult to evaluate.

We consider two different methods for reducing the computational burden in the optimal estimator. The first method is to replace $I(\hat{\delta})$ with a simpler function $I_L(\hat{\delta})$ whose maxima are in the same location as the maxima of $I(\hat{\delta})$. If $I_L(\hat{\delta})$ exhibits the same behavior as $I(\hat{\delta})$, then the locations of the global maxima of the two functions may well be the same too. This strategy still requires the computation of $I_L(\hat{\delta})$ at $N/3$ local maxima, but the computations are much simpler. The second method is to create a hybrid estimator using a combination of the maximum likelihood scheme of [2] with the optimum estimator. We consider these techniques next.

## III. Suboptimal Estimators

Let us again consider the conditional density function

$$h_{\mathbf{X}|\delta}(\mathbf{x}|k+\epsilon) = \phi(x_k - (1-\epsilon)\mu)\phi(x_{k+1} - \epsilon\mu) \prod_{\substack{i=0 \\ i \neq k,k+1}}^{N-1} \phi(x_i). \tag{5}$$

One suitable replacement for $h_{\mathbf{X}|\delta}(\mathbf{x}|u)$ is $h_L(\mathbf{x}, u)$, which we define to be the bounded and continuous piecewise linear function with endpoints $(k, x_k)$. Note that, as in [2], it can be easily shown that the maxima of the function

$$I_L(\hat{\delta}) = \int_{\hat{\delta}-1/2}^{\hat{\delta}+1/2} h_L(\mathbf{x}, u)\, du \tag{6}$$

are exactly the $\hat{\delta}_l$'s given in (4) . Hence, $\delta_L$, the global maximum of $I_L(\hat{\delta})$, is just one of the local maxima of $I(\hat{\delta})$, and it is our fondest hope that this local maximum of $I(\hat{\delta})$ is also its global maximum $\delta_{opt}$. This will certainly be the case when $\mu$ is small for the following reason. The Taylor series expansion of $h_{\mathbf{X}|\delta}(\mathbf{x}|u)$ around $\mu = 0$ is

$$\begin{aligned} h_{\mathbf{X}|\delta}(\mathbf{x}|u) &= [1 + \mu[(1-\epsilon)x_k + \epsilon x_{k+1}] + \ldots] \prod_{i=0}^{N-1} \phi(x_i) \\ &= [1 + \mu h_L(\mathbf{x}, u) + \ldots] \prod_{i=0}^{N-1} \phi(x_i). \end{aligned}$$

Thus, in the limit as $\mu \to 0$, $I(\hat{\delta}) \to [1 + \mu I_L(\hat{\delta})] \prod_i \phi(x_i)$, and hence the global maxima of $I(\hat{\delta})$ and $I_L(\hat{\delta})$ occur at the same point. In other words, $I_L(\hat{\delta})$ is just a linear function of the small-signal approximation for $I(\hat{\delta})$, and the estimation scheme proposed below is just the limiting form of the optimal estimator as the SNR $\mu$ approaches zero.

Substitution of $h_L(\mathbf{x}|u)$ for $h_{\mathbf{X}|\delta}(\mathbf{x}|u)$ allows us to derive an estimation scheme that is much simpler than $\mathcal{S}_{opt}$. As

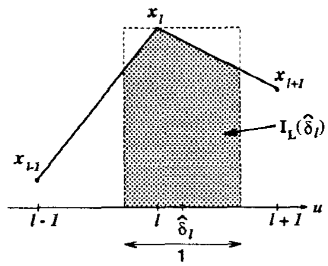


Figure 1: The statistic $I_L(\hat{\delta}_l)$ is the shaded area.

shown in Fig. 1, $h_L(\mathbf{x}|u)$ is a piecewise linear function, and hence simple geometry suffices to derive the following estimation scheme $\mathcal{S}_L$.

> For each $0 \le l \le N-1$, if $x_l > \max\{x_{l-1}, x_{l+1}\}$, there is a local maximum of $I_L(\hat{\delta})$ at
>
> $$\hat{\delta}_l = l + \frac{x_{l+1} - x_{l-1}}{2(2x_l - x_{l-1} - x_{l+1})}. \qquad (7)$$
>
> Compute
>
> $$I_L(\hat{\delta}_l) = x_l - \frac{(x_l - x_{l-1})(x_l - x_{l+1})}{2[2x_l - x_{l-1} - x_{l+1}]} \qquad (8)$$
>
> at each local maximum and find the global maximum of these values. $\delta_L$ is the location of the global maximum.

Since $\mathcal{S}_L$ is the limiting form of the optimal estimator for small SNR, we can expect its performance to be quite comparable to that of the optimal estimator for small SNR. In fact, simulation results (presented later in this paper) show that the performance is very good over a wide range of SNR. In addition, this scheme requires considerably less computation than the optimum scheme. Although $I_L(\hat{\delta})$ must be computed at each of the roughly $N/3$ local maxima, computation of (8) is almost trivial compared to that of (3) . Unfortunately, analytical results for the performance of this scheme remain quite difficult to obtain.

Although the computational requirements for $\mathcal{S}_L$ are considerably less than those for $\mathcal{S}_{opt}$, it is still necessary to compute $I_L(\hat{\delta})$ at numerous values of $\hat{\delta}$. We now consider an even simpler scheme which requires very little computation. Note from Fig. 1 that $I_L(\hat{\delta})$, which is the area of the shaded pentagon, can be approximated roughly (actually upper bounded) by the area of the rectangle of height $x_l$ on the unit base. Hence, we might choose to compare these approximations of $I_L(\hat{\delta})$ rather than the actual values of $I_L(\hat{\delta})$ in order to find the location of the global maximum. However, in doing this, the search is simplified considerably because it is no longer necessary to find the locations of all the local maxima first. Our scheme (which we denote by $\mathcal{S}_{mo}$) simply is

> Let $l^* = \arg\max_{0 \le l \le N-1}\{x_l\}$. Then the estimate of $\delta$ is
>
> $$\delta_{mo} = l^* + \frac{x_{l^*+1} - x_{l^*-1}}{2(2x_{l^*} - x_{l^*-1} - x_{l^*+1})}. \qquad (9)$$

$\mathcal{S}_{mo}$ is a hybrid scheme in the following sense. Consider the maximum likelihood scheme $\mathcal{S}_{mle}$ of [2] according to which a local maximum exists in $[l, l+1]$ if $|x_{l+1} - x_l| < \mu$ or if $x_l \ge \max\{x_{l-1}, x_{l+1}\} + \mu$. As the SNR $\mu$ approaches zero, the probability that the first condition is satisfied approaches zero. On the other hand, the second condition reduces to $x_l \ge \max\{x_{l-1}, x_{l+1}\}$, in which case the maximum is $2\mu x_l - \mu^2$ at $\hat{\delta}_l = l$. Thus, the limiting form of $\mathcal{S}_{mle}$ is $\delta_{mle} = l^* = \arg\max\{x_l\}$. The scheme $\mathcal{S}_{mo}$, however, differs in that it uses the right side of (9) as the location of the global maximum. Note that the right side of (9) is one of the $\hat{\delta}_l$'s of (4) that are the local maxima of $I(\hat{\delta})$. Thus, $\mathcal{S}_{mo}$ can be viewed as a hybrid of the limiting form of the maximum likelihood scheme with $\mathcal{S}_{opt}$ or $\mathcal{S}_L$. It also has the advantage that it requires very little calculation compared to either $\mathcal{S}_{opt}$ or $\mathcal{S}_L$ or even $\mathcal{S}_{mle}$ or $\mathcal{S}_{lo}$ [2]. Furthermore, because the decision statistic is so simple, we can actually compute its performance exactly, and these results can be used as approximations and bounds on the performance of the other schemes. Such analysis is considered next.

## IV. Performance Analysis

In general, analytical expressions for the error probabilities for most of the acquisition schemes are difficult to obtain. In [2] some bounds and approximations were found for the performance of $\mathcal{S}_{opt}$ and $\mathcal{S}_{lo}$. A rather loose lower bound on the conditional error probability (given $\epsilon$) that applies to the schemes presented in this paper is given in [3]. For $\mathcal{S}_{mo}$, however, we can derive an exact expression for the conditional error probability as follows.

Without loss of generality, let $\delta = 1 + \epsilon$. Then, $\mathcal{S}_{mo}$ acquires successfully if and only if either

1. $x_1 = \max_i\{x_i\}$ and $\left|\frac{x_2 - x_0}{2(2x_1 - x_0 - x_2)} - \epsilon\right| \le \frac{1}{2}$, or
2. $x_2 = \max_i\{x_i\}$ and $\left|\frac{x_3 - x_1}{2(2x_2 - x_1 - x_3)} + 1 - \epsilon\right| \le \frac{1}{2}$.

Recall that when $\delta = 1 + \epsilon$, the $X_i$'s are independent unit variance Gaussian random variables, and all have zero mean except for $X_1$ and $X_2$, which have means $(1-\epsilon)\mu$ and $\epsilon\mu$ respectively. Let $\bar{\epsilon} = 1 - \epsilon$. The probability that $\left|\frac{X_2 - X_0}{2(2X_1 - X_0 - X_2)} - \epsilon\right| \le \frac{1}{2}$, given that $X_1 = x$ and $X_2 = y$ and $X_1 > \max\{X_2, X_3\}$, is $\Phi(((1-2\epsilon)x + \epsilon y)/\bar{\epsilon}) = \Phi_1(x, y, \epsilon)$. The probability that $\left|\frac{X_3 - X_1}{2(2X_2 - X_1 - X_3)} - \epsilon\right| \le \frac{1}{2}$, given that $X_2 = x$ and $X_1 = y$ and $X_2 > \max\{X_1, X_3\}$, is $\Phi(((2\epsilon - 1)x + \bar{\epsilon}y)/\epsilon) = \Phi_2(x, y, \epsilon)$. Thus it is easily seen that

$$P_{e|\epsilon}[\mathcal{S}_{mo}] = 1 - \int_{-\infty}^{\infty}\int_{-\infty}^{x}\Phi(x)^{N-3}\phi(x-\bar{\epsilon}\mu)\phi(y-\epsilon\mu)\Phi_1(x,y,\epsilon)dy\,dx - \int_{-\infty}^{\infty}\int_{-\infty}^{x}\Phi(x)^{N-3}\phi(x-\epsilon\mu)\phi(y-\bar{\epsilon}\mu)\Phi_2(x,y,\epsilon)dy\,dx, \quad (10)$$

since the first integral is the probability that condition 1 holds, and the second integral is the probability that condition 2 holds. This expression can be evaluated numerically. The average value of this error probability over all $\epsilon$ from 0 to 1 is difficult to calculate, but we can bound the average error probability by finding the values of $\epsilon$ for which the conditional error probability is maximized and minimized. The derivative of (11) with respect to $\epsilon$ is zero at $\epsilon = 0.5$, and since the right hand side of (11) is symmetric about $\epsilon = 0.5$, we do have a local maximum or minimum at that point. Numerical evidence (e.g. Fig. 5) supports our assertion that $\epsilon = 0.5$ is indeed the location of the maximum value of (11) . Therefore, we conjecture an upper bound on the average error probability to be $P_{e|0.5}$, i.e.,

$$P_e[\mathcal{S}_{mo}] \leq 1 - 2\int_{-\infty}^{\infty}\int_{-\infty}^{x}\Phi(x)^{N-3}\phi(x-\mu/2) \times\phi(y-\mu/2)\Phi(y)\,dy\,dx. \quad (11)$$

This of course would also be an upper bound on the average error probability for $\mathcal{S}_{opt}$. Note that for $\epsilon = 0$, the second integral in (11) is zero and we have

$$P_{e|0} = 1 - \int_{-\infty}^{\infty}\Phi(x)^{N-1}\phi(x-\mu)\,dx, \quad (12)$$

which is the error probability for $N$-ary orthogonal signaling. Numerical evidence (e.g. Fig. 5) supports the conjecture that (11) is minimum at $\epsilon = 0$, so the right hand side of (12) is a conjectured lower bound on the average error probability for $\mathcal{S}_{mo}$.

The conditional error probability given by (11) involves time-consuming double numerical integration. A somewhat simpler formula that approximates (11) may be arrived at by assuming that the events in condition 1 that are necessary for $\mathcal{S}_{mo}$ to acquire successfully, i.e., $x_i = \max_i\{x_i\}$ and $\left|\frac{x_2-x_0}{2(2x_1-x_0-x_2)} - \epsilon\right| \leq \frac{1}{2}$, are independent, and likewise for the events in condition 2. This assumption leads to the following approximation for the conditional error probability of $\mathcal{S}_{mo}$ given $\epsilon$:

$$P_{e|\epsilon}[\mathcal{S}_{mo}] \approx 1 - \Phi(\mu\sqrt{(3\epsilon^2 - 3\epsilon + 1)/2})\int_{-\infty}^{\infty}\Phi(x)^{N-2} \times[\Phi(x-\epsilon\mu)\phi(x-\bar{\epsilon}\mu) + \Phi(x-\bar{\epsilon}\mu)\phi(x-\epsilon\mu)]dx. \quad (13)$$

Numerical results show that for several values of $\epsilon$ this formula approximates (11) quite well.

## V. Numerical Results

Because the error probabilities for most parallel acquisition schemes are very difficult to calculate, we must rely on Monte Carlo simulation to gauge their performance. Therefore, we have simulated the schemes $\mathcal{S}_{opt}$ and $\mathcal{S}_L$ using a length 1023 PN sequence, and used (11) to calculate the conditional error probability for $\mathcal{S}_{mo}$. Conditional error probabilities are determined for three values of $\epsilon$ and a range of values of $\mu$, which is a measure of the signal-to-noise ratio.

Fig. 2 is a plot of the simulated error probabilities and of (11) along with the approximation (13) as a function of $\mu$ for $\epsilon = 0.125$. All of the error probabilities are close to each other, and the approximation is practically indistinguishable from the actual error probability for $\mathcal{S}_{mo}$. The error probabilities in this figure are such that $P_{e,0.125}[\mathcal{S}_{mo}] < P_{e,0.125}[\mathcal{S}_L] < P_{e,0.125}[\mathcal{S}_{opt}]$. This is not a contradiction, because although $\mathcal{S}_{opt}$ has the smallest *average* error probability over all values of $\epsilon$, it may not necessarily have the smallest conditional error probability for every value of $\epsilon$. In fact, since the decision statistic of $\mathcal{S}_{mo}$ is the optimum one if $\epsilon = 0$, we expect $\mathcal{S}_{mo}$ to perform well for small values such as $\epsilon = 0.125$.

Fig. 3 is a plot of the error probabilities and (13) as a function of $\mu$ for $\epsilon = 0.25$. Again, the error probabilities of the three schemes are fairly close, but the order is now $P_{e,0.25}[\mathcal{S}_L] < P_{e,0.25}[\mathcal{S}_{opt}] < P_{e,0.25}[\mathcal{S}_{mo}]$.

Fig. 4 shows the error probabilities and (13) as a function of $\mu$ for $\epsilon = 0.5$. In this case, $\mathcal{S}_L$ still tracks the performance of $\mathcal{S}_{opt}$ fairly well, whereas $P_{e,0.5}[\mathcal{S}_{mo}]$ does not fall off quite as fast as $\mu$ increases. Note that this plot shows the (conjectured) worst case error probability (11) for $\mathcal{S}_{mo}$.

Fig. 5 shows the conditional error probability of $\mathcal{S}_{mo}$ as a function of $\epsilon$ for the particular value of $\mu = 8.0$. We believe this plot to be typical of the behavior of equation (13) for any $\mu$ in that the maximum value is at $\epsilon = 0.5$ and the minimum is at $\epsilon = 0$ and $\epsilon = 1$.

From the numerical results, we see that the suboptimal scheme $\mathcal{S}_L$ performs very well for the values of $\epsilon$ that we examined, and that $\mathcal{S}_{mo}$ does not perform significantly worse than the other schemes. The approximation we have given for $P_{e|\epsilon}[\mathcal{S}_{mo}]$ is also shown to be quite good.

## VI. Conclusion

In this paper, we have investigated suboptimal parallel acquisition schemes for PN sequences in DS/SS systems. The suboptimal schemes are based on the idea of replacing the conditional probability density function of the observations given the delay $\delta$ by the piecewise linear function connecting the observations in order to find simpler decision statistics. The scheme $\mathcal{S}_L$ performs very well and is considerably less costly to implement than $\mathcal{S}_{opt}$. However, it still involves significant computation. On the other

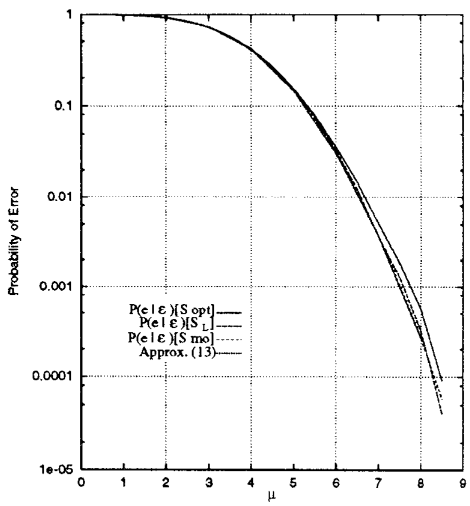


Figure 2: Error Probabilities for $\epsilon = 0.125$

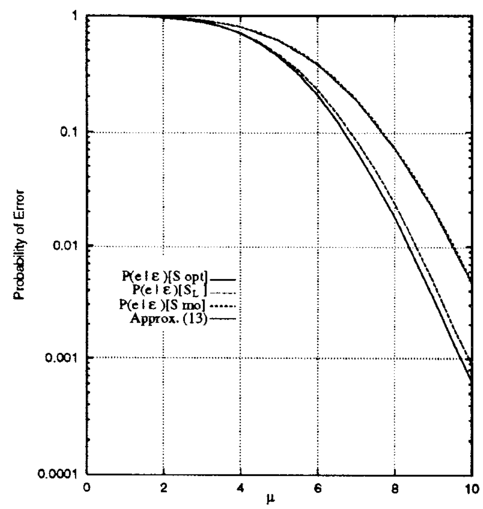


Figure 4: Error Probabilities for $\epsilon = 0.5$

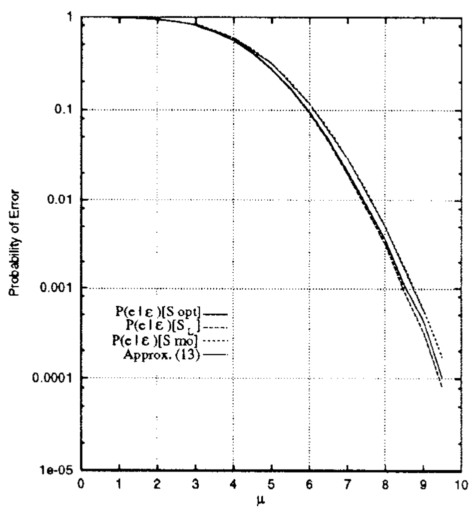


Figure 3: Error Probabilities for $\epsilon = 0.25$

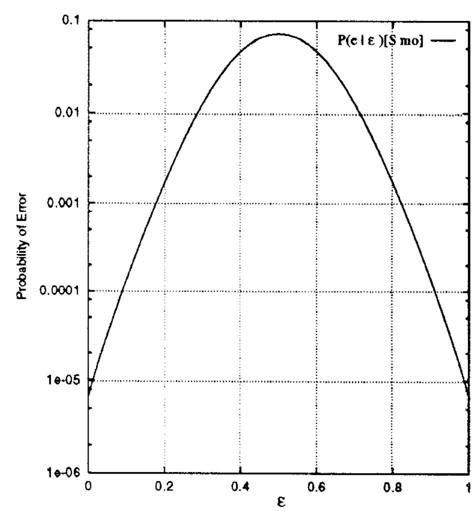


Figure 5: Error Probability of $\mathcal{S}_{mo}$ for $\mu = 8.0$

hand, the scheme $\mathcal{S}_{mo}$ degrades slightly in performance compared to $\mathcal{S}_L$ and $\mathcal{S}_{opt}$ for some values of $\epsilon$, but is very simple to implement. In fact, $\mathcal{S}_{mo}$ requires practically the minimum amount of computation and its performance can be determined analytically. A good approximation to the conditional error probability for $\mathcal{S}_{mo}$ is given, along with conjectured bounds on the average error probability. An additional asset of $\mathcal{S}_{mo}$ (as well as of $\mathcal{S}_L$) is the fact that one does not need an estimate of the SNR $\mu$ in order to implement the scheme. In contrast, both $\mathcal{S}_{opt}$ and $\mathcal{S}_{mle}$ require the value of $\mu$ for their estimates. Therefore, we believe $\mathcal{S}_{mo}$ is an excellent candidate for implementation.

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